06-720 Advanced Process Systems Engineering

04/30/04

Exam II

This 120 minute exam consists of five unequally weighted parts. Please budget your time carefully. You may consult your class notes, textbooks and homework sets to solve these problems.

1. Consider the Runge-Kutta method given by the following Butcher block:

$$\begin{array}{c|cc} a & a & 0 \\ 1 & b & a \\ \hline & b & a \end{array}$$

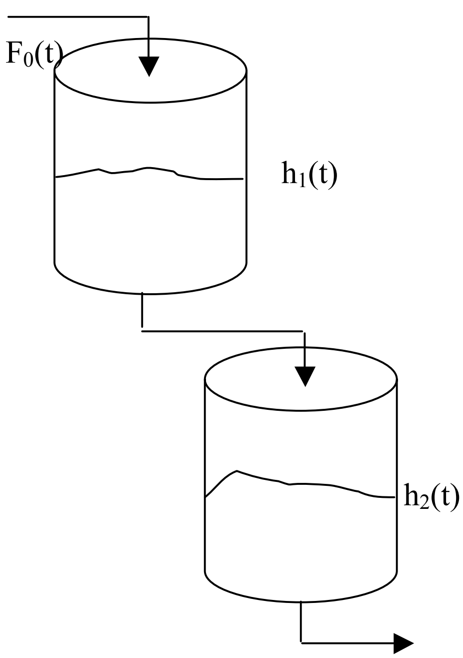
with $1 > a > b > 0$.

a) Write the Runge-Kutta formula. Is it implicit or explicit? (5 points)
b) What is the characteristic equation for stability for this method? What is the root of the difference equation? (5 points)

2. Consider the system

$$\begin{array}{ll} y_1' = -0.1\, y_1 - 0.5\, y_2 & y_1(0) = 0.2 \\ y_2' = -6\, y_2 & y_2(0) = 1 \\ y_3' = 70\, y_2 - 1200 y_3 & y_3(0) = 30 \end{array}$$

a) Calculate the eigenvalues and the stiffness ratio for this system. (10 points)
b) Find the maximum value for which h is stable for an explicit Euler method. (5 points)
c) Find the maximum value for which h is stable for a third order BDF method. (5 points)



3. Consider the two tanks in the figure with levels h_1 and h_2 for each tank and cross sectional areas of A_1 and A_2. The inlet flowrate to the first tank is $F_0(t)$ and the outlet flows for tanks 1 and 2 are given by $C_V\, (h_1)^{1/2}$ and $C_V\, (h_2)^{1/2}$, respectively.

a) Formulate a DAE system to find the inlet flow so that the two tank levels are equal at all points in time. What is the index of this system? (10 points)

b) Reformulate this system as an index 1 system that prevents drift from the algebraic constraints. (10 points)

(over, please)

4. For the system

$$y'' + y' + y = 0; \qquad y(0) = -1,\ y'(1) = 0$$

a) Describe a detailed single shooting method based on first order ODE's and sensitivity equations. Show all equations but do not solve. (10 points)

b) Using a power series basis apply two point orthogonal collocation and write the collocation equations. What are the collocation points? (10 points)

5. Consider a nonlinear batch reactor example with A → B → C, temperature as the control variable and rate constants $\alpha(T)$ and $\beta(T)$. It is desired to maximize an intermediate product after a variable reaction time. The problem is nonlinear in the rate equations for the concentration of A. Letting a(t) and b(t) represent the concentration of A and B, respectively, the optimal control problem is:

$$\begin{aligned}
&\text{Max} \quad b(t_f) \\
\text{s.t} \quad & a' = -\alpha(T)\, a \\
& b' = \alpha(T)\, a - \beta(T)\, b^2 \\
& a(0) = 1,\ b(0) = 0 \\
& T_l \le T(t) \le T_u
\end{aligned}$$

a) Write the Euler-Lagrange equations for this system *including an expression for optimal final time*. (10 points)
b) Describe a sequential strategy with an optimal control profile, optimal final time and sensitivity equations. (10 points)
c) Formulate the above optimal control problem as a nonlinear programming problem using collocation on finite elements. (10 points)